Analysis by the thermodynamic formalism for an experiment of an electronic circuit

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An experiment of a nonlinear RLC electronic circuit that exhibits chaotic behavior is analyzed by the thermodynamic formalism method. The band merging is ubiquitous in physical systems that show chaotic behavior. A dynamical variable of the experimental data just after the band merging is studied. The results show that a dynamic scaling law holds in the experimental data. The decay rate of temporal correlation has the scaling form as $\gamma_q = \kappa g(q/\kappa)$, where q is the parameter that characterizes the fluctuation of dynamical variable and κ is its characteristic value. The present analysis confirms that the thermodynamic formalism and the generalized power spectrum can be successively applied for time series obtained from concrete experiments. [S1063-651X(96)09205-7]

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I. INTRODUCTION

In recent years the thermodynamic formalism has been used in many disciplines [1,2]. It is very useful to characterize the self-similar statistics of physical systems. The first theoretical work describing the thermodynamic formalism [3] was published by Donsker and Varadhan, where it was presented as the *large deviation theory* [4–6]. On the other hand, the same framework was completed for physical systems [7–9] independently of the probability theory. This formalism is especially useful in characterizing nonlinear systems, and the usefulness of this approach has been presented already [1]. The use of the artificial parameter q corresponding to the temperature in thermodynamics makes it possible to investigate the statistical aspect of dynamical variables.

Recently this formalism has been extended to the order-q power spectrum in order to single out temporal correlations contained in the dynamical variable from the viewpoint of the thermodynamic formalism [10]. The order-q power spectrum is a theoretical approach to give a dynamic characterization of the formalism, in which various correlations of the dynamical variable can be extracted [11–14]. The ordinary power spectrum can capture the correlation of the dynamical variable, but it cannot determine, however, the property of the intermittency clearly. The order-q power spectrum can determine, on the other hand, the correlations of the laminar part and the burst part of the intermittency separately in a clear-cut way by changing the parameter q [12]. The determination is possible because the order-q power spectrum is a weighted power spectrum and the weight stresses a particular region of the time series of the dynamical variable. In spite of its importance the order-q power spectrum is not common for physicists except for the earnest supporters. Furthermore, it has not been used to analyze experimental data. The ordinary power spectrum is very popular for various experiments. At present, the order-q power spectrum is used only for the analyses of simple analytical models or computational analyses [11,12,14]. In this paper the analysis with the order-q power spectrum for the data obtained from an experiment of a nonlinear RLC circuit will be reported. In the past many temporal correlations might be abandoned in the conventional analysis, whereas these correlations involve essential features for physics. This paper contributes to nonlinear physics a way to extract various correlations or modes of motion from experimental data.

This paper is organized as follows. In Sec. II an experiment that produces a chaotic time series is introduced. This experiment was carried out with an electric circuit known as the driven nonlinear RLC circuit. In Sec. III first the thermodynamic formalism and the order-q power spectrum $I_q(\omega)$ are reviewed briefly. Next the experimental data are analyzed from the viewpoint of the thermodynamic formalism, and the order-q power spectrum $I_q(\omega)$ is used to investigate the data obtained from the experiment. Concluding remarks are given in Sec. IV.

II. AN EXPERIMENT FOR AN ELECTRONIC CIRCUIT

To test our approach we will use a simple electronic circuit exhibiting chaos [15,16]. The experimental arrangement is shown schematically in Fig. 1; in this case the nonlinear element in the circuit is the diode. The values of the inductance and the resistances are as follows: $L=105.3$ μH, $R=10.3$ Ω, and $r=50$ Ω, where r is the output impedance of an oscillator. The diode is of type 1N4004. The driving voltage $E(t)$ is a function of t as $E(t)=A\cos 2\pi\nu t$. This experimental arrangement shows the chaotic behavior involving the band merging ubiquitous in chaos [15]. When we vary the input frequency ν, we can observe the band merging behavior. The phase portraits of $V(t)$ versus $V_0(t)$ taken on to the oscilloscope screen are shown in Fig. 2, where $V(t)$ is the voltage across the diode and $V_0(t)$ the voltage across the diode and the resistor R.

In this experiment $V(t)$ is taken into the digital oscilloscope with 0.1-μs sampling time, where values of $V(t)$ are digitized into 250 points. The original sampling data are supplemented by the spline function to find precise extrema, which will be analyzed later. Just after the band merging, extrema alternately take two nearby values and phase jumps

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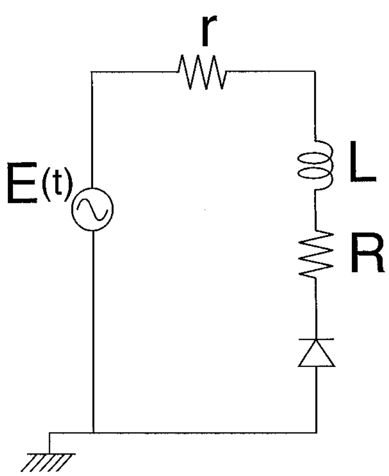

FIG. 1. Driven nonlinear RLC circuit to produce chaotic time series.

sometimes occur. Namely, the experimental result shows the so-called intermittent switching that the time series dwells in one band for a while and then intermittently switches to the other [17]. We investigate the intermittent switching observed in this experiment. Recently it has been shown that in some mathematical models the time series that presents the intermittent switching have the static and dynamic scaling laws [11,18,19].

In this paper we will confirm the existence of scaling laws in the results obtained from the electronic experiment. The thermodynamic formalism and the order-q power spectrum are used to analyze the experimental data. The $f(\alpha)$ spectrum [8] in the multifractal, which has the same principles as the thermodynamic formalism, has been used to characterize the forms or patterns of the complex systems in experiments. Our results will show how useful the thermodynamic formalism and the order-q power spectrum are and how to apply them to analyze the real systems.

III. ANALYSIS BY THE ORDER-*q* POWER SPECTRUM

In this section we briefly review the thermodynamic formalism following Ref. [11]. Let us take a time series $\{u_j\}_{j=1}^{nN}$, which is composed of N subregions. In the experiment reported in this paper this series will be the voltage across the diode. The kth subregion ($k=1,2,\ldots,N$) has the average value

$$u_n\{k\}=\frac{1}{n}\sum_{j=1}^{n} u_{j+(k-1)n}. \tag{1}$$

It should be noted that the averaging extent n is large enough in comparison with the characteristic time of the system. The probability density that u_n takes a value u' is represented by

$$P_n(u')\equiv\langle\delta(u_n-u')\rangle=\lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}\delta(u_n\{k\}-u'), \tag{2}$$

where $\langle\cdots\rangle$ means the ensemble average and $\delta(\,)$ is the Dirac distribution. In many systems $P(u')$ asymptotically takes the form

$$P_n(u')\sim e^{-nS(u')} \tag{3}$$

for large n [6,20]. This asymptotics is valid only when n is taken to be larger than the largest characteristic time of the system. The entropy function $S(u')$ ($\geqslant 0$) shows how the fluctuation of $u_n\{k\}$ from the long time average u_∞ reduces. Here we introduce the free energy function $\Phi(q)$ by

$$M_q(n)\equiv\langle\exp(nqu_n)\rangle\sim e^{\Phi(q)n}, \tag{4}$$

where

$$\Phi(q)=\lim_{n\to\infty}\frac{1}{n}\ln M_q(n). \tag{5}$$

The functions $S(u)$ and $\Phi(q)$ are related via the Legendre transform

$$u(q)=\frac{d\Phi(q)}{dq},\quad S(u)=qu-\Phi(q),\quad q=\frac{dS(u)}{du}. \tag{6}$$

We add the susceptibility function $\chi(q)$ that is defined as

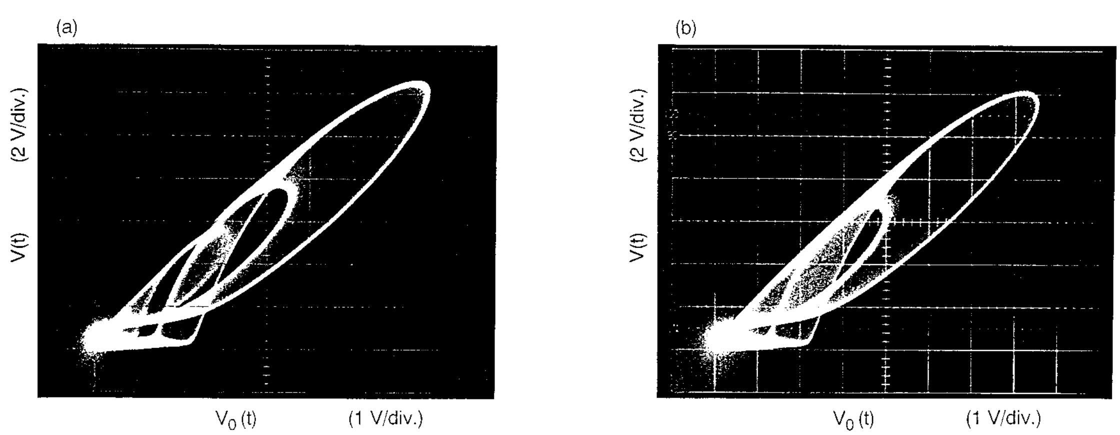

FIG. 2. Bands on the oscilloscope for $V_0(t)$ vs $V(t)$ before the band merging (a) and after the band merging (b). The change from (a) to (b) happens as the value of the control parameter ν, the input frequency, is increased.

$$\chi(q) \equiv \frac{du(q)}{dq}. \tag{7}$$

Various time correlations are represented by the order-q power spectrum $I_q(\omega)$ defined as [10]

$$I_q(\omega) \equiv \lim_{n\to\infty} \langle F_n(\omega) e^{qnu_n} \rangle / M_q(n), \tag{8}$$

where

$$F_n(\omega) \equiv \left| \frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} (u_j - \langle u_j \rangle) e^{-i\omega j} \right|^2 \tag{9}$$

is the Fourier spectrum. The order-q power spectrum is the ordinary power spectrum obtained from the time series whose average is $u(q)$. Before we present the results given by the computational calculation of the data, we will discuss the approach we have used. We have used a phenomenological approach in which the properties of the intermittent switching are captured.

The key idea in later discussion is that the intermittent switching can be approximately considered as a Markov process, where the dynamical variable takes two values. Let v_i ($i=1,2$) be the observed value in the ith state and $p_i(j)$ be the probability that the dynamical variable is in the ith state at time j. The Markov process is described by

$$P(j+1) = HP(j), \tag{10}$$

where $P(j) = \binom{p_2(j)}{p_2(j)}$. H is the transition matrix composed of transition probabilities and the steady-state distribution P_* is given by the equality

$$P_* = HP_*. \tag{11}$$

On the basis of this Markov process, we calculate the thermodynamic functions and $I_q(\omega)$. Let us introduce the generalized transition matrix H_q given by

$$H_q = \begin{bmatrix} h'_{21} e^{-vq} & h_{12} e^{vq} \\ h_{21} e^{-vq} & h'_{12} e^{vq} \end{bmatrix} \quad (h'_{ij} \equiv 1 - h_{ij}), \tag{12}$$

where we set $v_1 = -v$ and $v_2 = v$ without loss of generality. h'_{ij} and h_{ij} are transition probabilities. H_0 is identical to H in Eq. (10). The eigenvalues of H_q give thermodynamic functions and $I_q(\omega)$ [11]. The largest eigenvalue of H_q gives

$$\Phi(q) = \ln \frac{h'_{12} e^{vq} + h'_{21} e^{-vq} + (\{h'_{12} e^{vq} - h'_{21} e^{-vq}\}^2 + 4 h_{12} h_{21})^{1/2}}{2}, \tag{13}$$

which yields

$$u(q) = \frac{h'_{12} e^{vq} - h'_{21} e^{-vq}}{[(h'_{12} e^{vq} - h'_{21} e^{-vq})^2 + 4 h_{12} h_{21}]^{1/2}}, \tag{14}$$

$$\chi(q) = \frac{4 v^2 h_{12} h_{21} (h'_{12} e^{vq} + h'_{21} e^{-vq})}{[(h'_{12} e^{vq} - h'_{21} e^{-vq})^2 + 4 h_{12} h_{21}]^{3/2}}. \tag{15}$$

The order-q power spectrum is, on the other hand, given by [11]

$$I_q(\omega) = \frac{K_q \sinh(\gamma_q)}{2[\sinh^2[\gamma_q/2] + \sin^2[\omega/2]]}, \tag{16}$$

where

$$\gamma_q = \ln\left[\frac{h'_{12} e^{vq} + h'_{21} e^{-vq} + [(h'_{12} e^{vq} - h'_{21} e^{-vq})^2 + 4 h_{12} h_{21}]^{1/2}}{h'_{12} e^{vq} + h'_{21} e^{-vq} - [(h'_{12} e^{vq} - h'_{21} e^{-vq})^2 + 4 h_{12} h_{21}]^{1/2}} \right], \tag{17}$$

$$K_q = \frac{4 h_{12} h_{21}}{(h'_{12} e^{vq} - h'_{21} e^{-vq})^2 + 4 h_{12} h_{21}}. \tag{18}$$

Just after the band merging the transition probability between two states is sufficiently small, then we can derive the scaling forms for the thermodynamic functions. We set $h_{21} = \kappa$, $h_{12} = b\kappa$, and then take the limit $\kappa \to 0$. The limit $\kappa \to 0$ corresponds to the situation where the system is just after the band merging and the dwelling time in one band is very long. In the limit $q \to 0$ and $\kappa \to 0$ by keeping $x = q/\kappa$ finite, the scaling laws hold as

$$\chi(q) = \frac{1}{\kappa} f\left(\frac{q}{\kappa}\right), \tag{19}$$

$$\gamma_q = \kappa g\left(\frac{q}{\kappa}\right), \tag{20}$$

where the scaling functions are

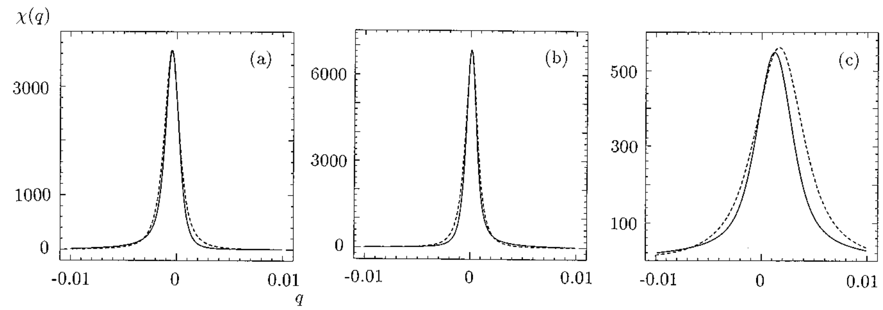

FIG. 3. The susceptibility function $\chi(q)$ obtained from the experimental data is expressed by solid lines at each value of the control parameter. The dashed lines are the fitted $\chi(q)$'s assuming the two-state Markov model. The distance from the merging point is nearest for (a), middle for (b), and farthest for (c).

$$f(x)=\frac{8v^2b}{[(2vx-b+1)^2+4b]^{3/2}}, \tag{21}$$

$$g(x)=[4v^2x^2+4v(1-b)x+(1+b)^2]^{1/2}. \tag{22}$$

In this experiment, the frequency of the driving force is chosen as the control parameter. The control parameter is changed monotonically as the distance from the merging point increases. We set it at 0.733, 0.740, and 0.755 MHz. The number of data values used for the present analysis is 1024×25 at each value of the control parameter. The averaging span n is set at 512. The thermodynamic function $\chi(q)$ at each value of the control parameter is shown in Fig. 3. In this case the parameter b has different values at three points of the control parameter. We determine κ and b to fit $\chi(q)$ calculated from Eq. (15) to the one obtained from the experimental data. First we define v by setting $v\equiv(V_2-V_1)/2$, where V_1 and V_2 are approximately minimal and maximal values of $u(q)$ obtained from the experimental data, respectively. It should be noted that we assume that V_1 and V_2 are temporary and the experimental data take various minimal and maximal values. Second we determine κ and b such that $\chi(0)$ and the maximum $\chi(\hat{q})$, $\hat{q}$ being the peak position of $\chi(q)$, obtained from the experimental data fit to those of the two-state Markov model as shown in Fig. 3. The values of κ and b determined in this manner are shown in Table I. The values of κ and b are recorded at three points of the control parameter.

Let us turn now to the dynamic scaling law. We determined κ and b at each value of the control parameter. From these two values, the peak width γ_q of $I_q(\omega)$ can be determined by assuming the two-state Markov model. This γ_q is the criterion that must be compared with the one calculated from $I_q(\omega)$ obtained from the experimental data via its definition Eqs. (8) and (9). The order-q power spectrum $I_q(\omega)$ for the experimental data and the fitted one are compared in Fig. 4. The scaling form of γ_q/κ can be written explicitly as a function of $x=q/\kappa$ using b and v as

$$\gamma_q/\kappa=\sqrt{4v^2x^2+4v(1-b)x+(1+b)^2}. \tag{23}$$

On the other hand, γ_q for the experimental data is determined by assuming that $I_q(\omega)$ obtained from the experimental data is written as in Eq. (16). In practice $I_q(\omega)$ obtained from the experimental data is assumed to fit that in Eq. (16). The decay rate γ_q of temporal correlations determined from the experimental data at one value of the control parameter is shown in Fig. 4. The values of γ_q/κ determined from the experimental data and the typical q dependences of γ_q/κ are shown in Fig. 5 with open circles and solid lines, respectively. These figures show qualitative agreement except for Fig. 5(c). This might mean that the experimental data just after the band merging have the scaling law and a self-similarity. But only in the region where the control parameter is farthest from the band merging point, the result does not show good agreement. This implies that the data have the scaling laws just after the band merging.

We have shown that the thermodynamic functions and the order-q power spectrum can be calculated from experimental data comparatively easily and the scaling law exists in the experimental data. The time series that exhibits the intermit-

TABLE I. Values of κ and b are determined such that $\chi(q)$ calculated from the two-state Markov model fits the one obtained from the experimental data. Values of v are given by the upper limits and the lower limits of $u(q)$'s. (a), (b), and (c) have the same values of the control parameter as in Fig. 3.

	κ	b	v
(a)	0.006 687 91	0.423 513	4.0
(b)	0.004 770 28	1.211 21	6.0
(c)	0.004 532 83	2.441 27	2.0

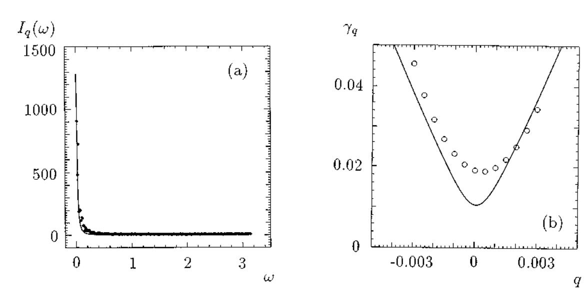

FIG. 4. $I_{q=0}(\omega)$ obtained in the experiment is expressed by dots in (a). The solid line in (a) is the fitted $I_{q=0}(\omega)$ by assuming the form Eq. (16). The open circles in (b) express the peak width γ_q of the fitted curve in (a). The solid line in (b) is γ_q obtained from the fitted $\chi(q)$ that is expressed by the dashed line in Fig. 3. The value of the control parameter in these figures is the same as in Fig. 3(b). The unit of ω is radian.

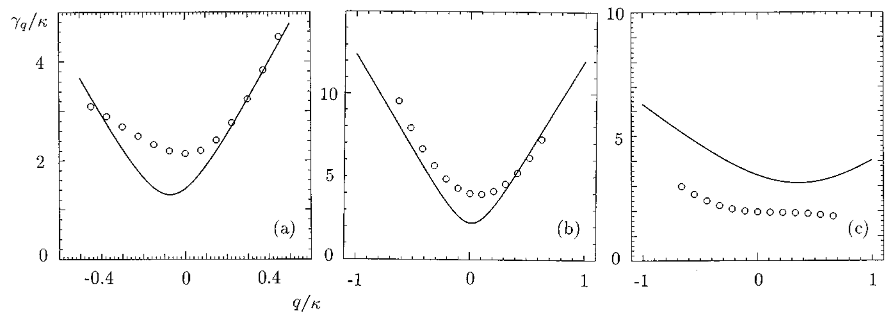

FIG. 5. γ_q/κ vs q/κ is expressed by open circles that are obtained directly from the experimental data by assuming Eq. (16). The solid lines are obtained from the fitted $\chi(q)$'s shown with dashed lines in Fig. 3. (a), (b), and (c) have the same values of the control parameter as in Fig. 3.

tent switching has scaling laws, which had been previously verified only for mathematical models, for example, logistic map, double-well potential system, parametrically excited pendulum, etc. [11]. We believe our results have confirmed an important point regarding the existence of a scaling law extracted directly from the experimental data.

IV. CONCLUDING REMARKS

The chaotic time series generated by an experiment for the forced *RLC* electronic circuit was studied by the thermodynamic formalism and its extended form. Our results have shown that the thermodynamic formalism can be also applied to a simple experimental apparatus. In particular, for the chaotic time series the scaling law just after the band merging was confirmed, similarly to that in many mathematical models studied previously.

The results reported in this paper present a way to analyze the experimental time series that exhibits chaos. In experiments the ordinary power spectrum $I_{q=0}(\omega)$ is used commonly for measurements. But this gives only average modes of motion. Note that nature has various correlations that are not captured on the average. The order-q power spectrum can describe various correlation characteristics that cannot be singled out by the ordinary one. This is the important point supporting the use of the order-q power spectrum. The authors hope this paper will set a good example for the use of $I_q(\omega)$.

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